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The Memristive Pupil: A Memristive Circuit Model of the Eye's Response to Illumination Fluctuations

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***Abstract*—The memristor has shown great promise as a circuit component with which nonlinear biological systems may be modeled. While some researchers have undertaken the challenge of producing memristive circuits that mimic biological neurons, there exists a lack of established memristive circuitry for pupil modeling. This paper proposes a new model that mimics the eye's responses to light intensity fluctuations by means of a voltage-controlled memristive circuit. The proposed pupil model exhibits promising results when examined in relation to the properties of biological data.**

***Index Terms*—memristor, memristive system, nonlinear circuit, biological circuit, pupil, pupillography, pupillometry**

I. Introduction and Objectives

A. Background

Since the first description of the conceptual memristor, researchers have attempted to take advantage of the nonlinear behavior produced by this unique device. Thanks to its nonlinear and nonvolatile properties, the memristor has been used to model biological systems in ways not previously imagined [1]. While the memristor certainly lends itself well to neuron modeling, its dynamics provide the potential for use in many other nonlinear biological models [1], [2]. Although biological systems like the human pupil have been modeled mathematically [3], the concept of a memristive pupil model has yet to be well-established.

B. A Brief Discussion of Pupillography

To properly grasp the requirements of a pupil model, one must understand the basic principles of pupil behavior observed during a pupillographic experiment. Pupillography involves the study of the pupil's reaction to illumination changes. Since the invention of infrared video pupillographs, the act of studying the pupil's characteristics has become increasingly easier [4], [5].

When performing a pupillographic test, the subject is acclimated to a dark environment and then exposed to a brief burst of light. An infrared video camera tracks the diameter of the pupil during the test [4], [5], [6]. A pupillogram is then created by plotting the subject's pupil diameter with respect to time. This information can be used to determine the responsiveness of the subject's pupil and can help to identify conditions such as Horner syndrome [5]. The second figure provided in [5] serves as an excellent visual example of biological pupillary data.

C. Goals

This paper proposes a memristor-based pupil model that mimics the eye's response to changes in illumination. The stimulus and output are given as voltage values where the input voltage corresponds to light intensity and the output voltage corresponds to pupillary diameter. This configuration allows the model to produce results that can easily be compared to biological pupillograms. The proposed pupil model makes use of a memristor model with an asymmetric rate function to mimic pupillary responses. The pupil model is simulated and analyzed with regard to biological results.

II. Design and Methodology

A. The Memristor Model

The proposed model makes use of a simple memristor model defined in Verilog-A and simulated in the Cadence Virtuoso environment. This voltage-controlled memristor resembles an imbalanced linear ionic drift model with the threshold effect. The simplistic linear ionic drift model can be applied to assume a rate of memristance change (r_M) that is linearly related to the voltage on the memristor (V_m) [1].

The memristor model is defined with a memristance value that is unchanged when the voltage across the memristor lies in a specified range, i.e. $r_M(V_m) = 0$ when $V_{reset} \leq V_m \leq V_{set}$. The model's threshold values are defined as $V_{set} = 0.3$ V and $V_{reset} = -0.3$ V. The memristor's value is assigned lower and upper boundaries of 10 kΩ and 500 kΩ, respectively. The memristance value is initialized to 500 kΩ.

To achieve results that reflect those of biological pupils, the rate function r_M is defined such that the memristor model experiences a faster change in memristance when a positive voltage is applied than it does when a negative voltage of the same magnitude is applied. The constant memristance change rate (γ) is defined as $\gamma_1 = -2.5 \frac{\text{M}\Omega}{\text{V}\cdot\text{s}}$ for positive voltage values and $\gamma_2 = -20 \frac{\text{k}\Omega}{\text{V}\cdot\text{s}}$ for negative voltage values. The rate of change of memristance as a function of the voltage across the memristor can therefore be given as

$$r_M(V_m) = \begin{cases} (-2.00\,V_m - 0.60) \times 10^4 \ \Omega s^{-1} & \text{for } \ V_m < -0.3 \text{ and } M < 500 \text{ k}\Omega \\ (-2.50\,V_m + 0.75) \times 10^6 \ \Omega s^{-1} & \text{for } \ V_m > 0.3 \text{ and } M > 10 \text{ k}\Omega \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where V_m is the voltage across the memristor and M is the present memristance value.

This memristive model maintains a higher sensitivity to positive voltages than it does to negative voltages, i.e. it has a high γ_1/γ_2 ratio. This allows for a biologically resemblant pupil model that contracts faster than it dilates [3], [5]. When connected to a sinusoidal voltage source, the model's pinched hysteresis varies with successive periods due to the sensitivity imbalance. Fig. 1 presents this hysteretic behavior.

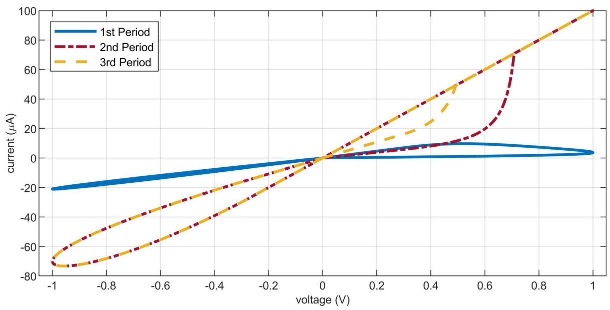

Fig. 1. Hysteresis behavior of the memristor model when connected to a 1 Hz sinusoidal voltage source with an amplitude of 1 V.

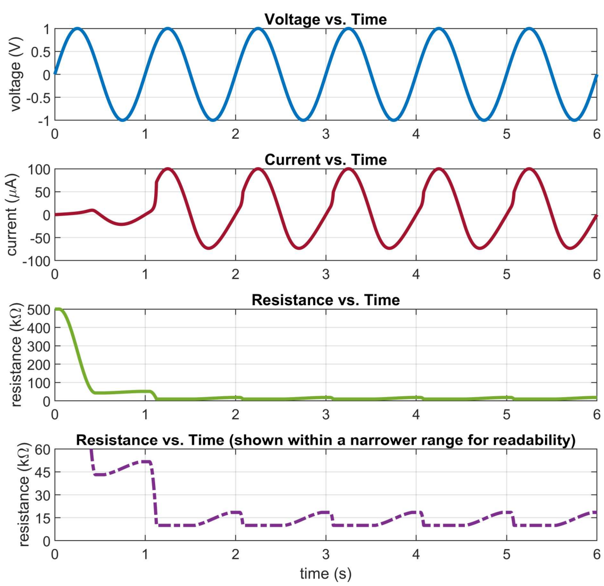

Fig. 2. Time domain behavior of the given memristor model when connected to a 1 Hz sinusoidal voltage source with an amplitude of 1 V.

As Fig. 2 demonstrates, the model's imbalance causes the memristance to decrease faster than it increases when connected to a sinusoidal voltage source. In this test, the memristance value becomes repetitive once the third period is reached. Additionally, the memristance value remains in a relatively narrow range beginning with the second period.

B. The Pupil System Model

The model of the pupil system is designed to be straightforward in terms of usage and adaptability. The input voltage signal ranges from 0 V to 1 V which corresponds to minimum and maximum illumination, respectively. The model's output ranges from 0.5 V to 0.98 V which corresponds to minimum and maximum pupil diameter, respectively. The output voltage is obtained by taking the ratio of two voltages within a voltage divider circuit consisting of a resistor and memristor.

To operate predictably, a relationship between input voltage and steady state output voltage was devised. For simplicity, the steady state output voltage is modeled as a linear function of input voltage. In other words, it is assumed that if the steady state pupil diameter is d_1 at illumination i_1 and the steady state pupil diameter is d_2 at illumination i_2, then the pupil diameter at $\frac{i_1+i_2}{2}$ can be given as the midpoint between d_1 and d_2 (equivalent to $\frac{d_1+d_2}{2}$). Using the previously defined input and output voltage ranges, the steady state output voltage (V_{ss}) as a function of input voltage is approximated as:

$$V_{ss} = -0.48v_{in} + 0.98 \tag{2}$$

The overall pupil model exists as a straightforward closed-loop design. The steady state output voltage is calculated from the input and is then subtracted from the present output voltage to produce an error value. This error value is amplified and sent through a low-pass filter to smooth possible voltage spikes. This filtered error value (V_r) is then sent to the voltage divider circuit. If the voltage magnitude exceeds a predefined threshold, it will alter the memristance value. This process is summarized in Fig. 3.

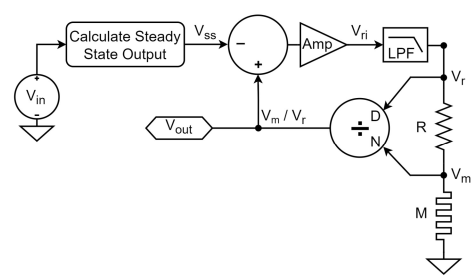

Fig. 3. Block diagram of the memristive pupil model

The output of the circuit is taken as a ratio of two voltages: $V_{out} = \frac{V_m}{V_r}$. The two voltages are taken from a simple voltage divider consisting of a resistor in series with a memristor. The resistance R was set to 10 kΩ (equal to the minimum resistance value of the memristor). As a result, the memristor voltage V_m lies in the range $0.5V_r \leq V_m \leq 0.98V_r$. Therefore, the value of the model's output ($\frac{V_m}{V_r}$) lies in the range $0.5 \leq V_{out} \leq 0.98$. It is for this reason that (2) is defined for an output in the approximate range of 0.5 V to 0.98 V. To summarize these relations, the output voltage as a function of memristance can be given as

$$V_{out}(M) = \frac{M}{R+M} = \frac{M}{10000+M} \tag{3}$$

1) Difference Amplifier for Calculating Steady State Output: The steady state voltage calculator is designed as a difference amplifier like the one shown in Fig. 4. The circuit acts as an analog implementation of the calculation in (2). The circuit output is given as $V_{out} = \left[\frac{R_4(R_1+R_3)}{R_1(R_2+R_4)}V_A - \frac{R_3}{R_1}V_B\right]$. To produce an output corresponding to (2), the resistance values of $R_1, R_2, R_3,$ and R_4 are set to 680 kΩ, 200 kΩ, 330 kΩ, and 390 kΩ, respectively. The model's input V_{in} is connected to V_B and a 1 V source is connected to V_A to produce the relationship $V_{ss} \approx -0.485v_{in} + 0.982$. This very closely approximates (2). A ±2 V supply powers the amplifier.

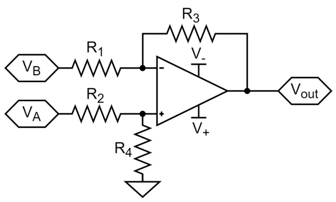

Fig. 4. Difference amplifier design

2) Difference Amplifier for Calculating Error: The second difference amplifier calculates and amplifies the difference between V_{out} and V_{ss}. The resistance values of $R_1, R_2, R_3,$ and R_4 are set to 100 kΩ, 100 kΩ, 10 MΩ, and 10 MΩ, respectively. This produces a total gain of 100 with a corresponding equation of $V_{out} = 100(V_A - V_B)$. Without this high gain, the magnitude of V_r and V_m would only rise above the memristor's threshold voltage when the error is extremely high. Having this high gain allows V_r and V_m to rise high enough to alter the memristance value when the error surpasses approximately ±3 mV.

3) Lowpass Filter: The lowpass filter is used to smooth sudden voltage spikes and is constructed as a simple RC lowpass filter. A 100 Ω resistor and a 4.7 µF capacitor are used to give the filter a cutoff frequency of about 340 Hz. This filter helps to mitigate the effects of spikes and improves simulation convergence times.

4) Division Block: The final stage of the model determines the output voltage by dividing V_m by V_r. This configuration was decided upon because the result is not affected by the polarity of the values and because (3) works well at simulating a natural pupil response. For simplicity, this division process was defined in Verilog-A code. If this entire pupil model were to be physically constructed, the division block could be implemented in the analog domain with a sophisticated division circuit or digitally with a basic microprocessor.

5) Overall System: The top-level schematic combines the previous subcircuits to produce the final model in Fig. 5.

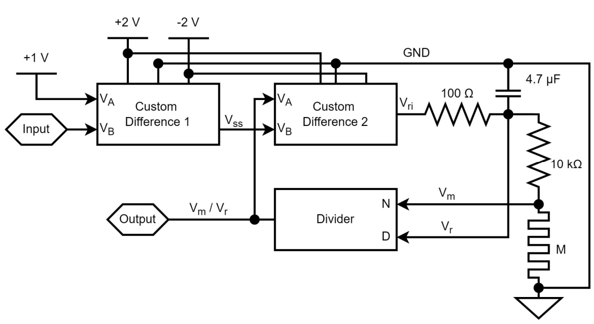

Fig. 5. Top-level schematic of the memristive pupil model

III. Results and Evaluation

A. Response to Light Pulses

All simulations were performed in the Cadence Virtuoso software environment. The first test was conducted in a manner similar to the biological test outlined in [5]. The test performed on the pupil model began with an input of 0 V (representing darkness) followed by one second of the maximum 1 V input (representing bright light) followed by 24 seconds at 0 V. The pulse was given a virtually instantaneous rise and fall time of 1 µs. Note that a long acclimation period is not needed for tests that begin with a 0 V stimulus because the model is initialized to steady state darkness conditions.

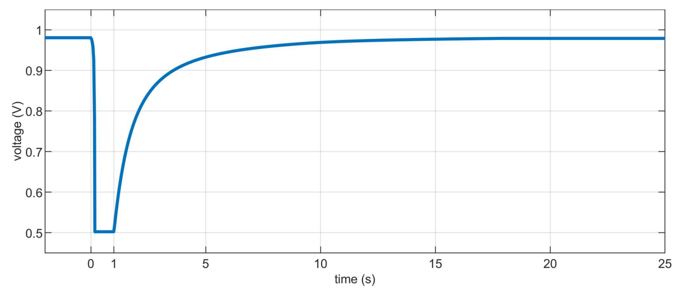

Fig. 6. Pupil model's response to a one-second, 1 V pulse occurring at t=0

The results of this light pulse test are shown in Fig. 6. The general shape of the plot largely resembles the biological test results shown in [5] as well as those obtained by [6] and [7]. The output voltage quickly drops in the presence of simulated light and slowly rises in response to simulated darkness. This behavior is primarily produced by the imbalance of the memristor model, i.e. the high γ_1/γ_2 ratio. The nature of the voltage divider circuit also allows for the gradual return to maximum pupil diameter (high output voltage) because the rate of change of V_{out} with respect to M declines as M increases, i.e. $\frac{d}{dM}V_{out}(M) = \frac{10000}{(10000+M)^2}$.

To demonstrate the repetitive nature of the model, a one-second pulse was periodically applied to the model. The model was acclimated to a 0 V stimulus and then exposed to one second of a 1 V input followed by nine seconds at 0 V before repeating the pattern. The results given in Fig. 7 show that the model does behave periodically when provided with a periodic stimulus. This steady contraction and dilation behavior is consistent with Ellis' biological results in [6].

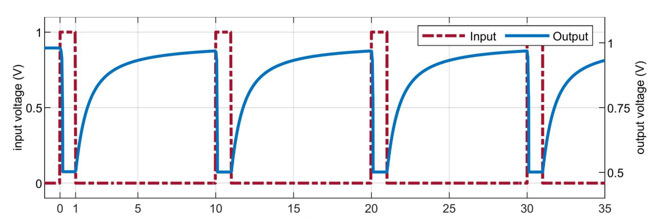

Fig. 7. Pupil model's response to a periodic one-second maximum pulse (10-second period with 10% duty cycle)

To ensure proper responses to intermediate V_{in} values, the model was tested with mid-range stimuli. The stimulus spent two seconds at 0.3 V (representing dim light) for acclimation purposes, rose to 0.7 V (representing moderately bright light) for one second, and then fell back to 0.3 V. As Fig. 8 shows, the model properly conformed to (2) by approaching the anticipated steady state outputs of 0.84 V and 0.65 V in the presence of 0.3 V and 0.7 V inputs, respectively. Since the stimuli range was smaller than that of the first test, the model took less time to reach steady state after the pulse. During the first test (using a 0-1 V pulse), the model took approximately 6.13 seconds to climb 95% of the way to its steady state value after the pulse. During this mild pulse test (0.3-0.7 V), the model took approximately 0.97 seconds to hit the 95% mark.

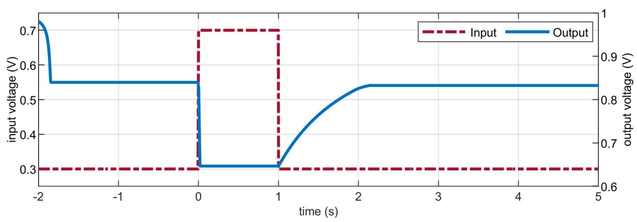

Fig. 8. Pupil model's response to a one-second mild pulse (0.7 V) at t=0 after acclimation to dim light (0.3 V)

B. Repose to Sinusoidal Sources

The final tests made use of a sinusoidal stimulus. The model was tested with both a 1 Hz and a 10 Hz sinusoidal source. The input waveform was given an amplitude of 0.5 V and an offset of 0.5 V. The waveforms at both frequencies are shown in Fig. 9. According to [5], the human pupil response to a sinusoidal stimulus "has a low pass characteristic" with a cutoff frequency of about 9 Hz. Once this cutoff is reached, the human pupil treats the light source more like a constant value than an oscillating one.

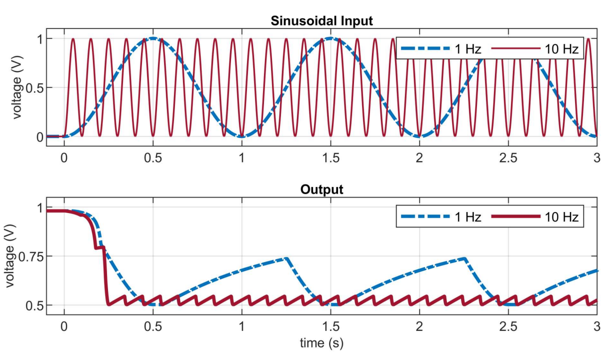

Fig. 9. Pupil model's response to sinusoidal waveforms with frequencies of 1 Hz and 10 Hz

The results shown in Fig. 9 demonstrate that the model produces a periodic response to both the 1 Hz and 10 Hz stimuli. This behavior stands out as a notable contrast to the biological pupil's 9 Hz lowpass cutoff [5]. Further development with the inclusion of a lowpass filter at either the output or input of the circuit could serve to minimize or resolve this discrepancy.

IV. Discussion of Results and Limitations

Overall, the model's behavior aligns with expectations regarding relative response times and the behavior set forth in (2). The asymmetry of the pupil's response correlates well with the biological results in [5] and [6]. The model demonstrates its effectiveness in responding to illumination fluctuations of various intensities. Furthermore, the model can be adapted to accommodate faster or slower pupillary responses by modifying the γ and R parameters.

When studying fine details, the model displays imperfections. The model does not account for the response latency observed in the biological pupils of [5] and [6]. Additionally, the model displays a plateauing behavior during the stimulus time frame which was not observed in the biological systems found in [5]. If the model were realized as a physical circuit, care would need to be taken to minimize noise and ensure proper component tolerances, especially considering the difference amplifier's high gain. Further development could mitigate these limitations and lead to a model that accounts for other response factors such as light wavelength or mental and muscle activity - all of which can affect pupil responses [3], [5], [8].

V. Conclusions

In light of the results, the memristive pupil model has demonstrated some of the properties of a biological pupil. The model manifests the asymmetric relations observed in biological tests and its reaction speed is influenced by the magnitude of the light intensity change. Furthermore, the model can be adapted to accommodate faster or slower responses by modifying the γ and R parameters. This opens up the possibility of creating more complex responses with various determinant factors. Overall, the simple structure provides room for refinement and allows for modifications to suit various purposes. With further development, the proposed memristive pupil model can be used to create an accurate and straightforward system that mimics the human eye in a novel and unconventional manner.

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